

ACCURACY, DEFERENCE, AND CHANCE

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ABSTRACT

Chance both guides our credences and is an objective feature of the world. How and why we should conform our credences to chance depends on the underlying metaphysical account of what chance is. I use considerations of accuracy (how close your credences come to truth-values) to propose a new way of deferring to chance. The principle I endorse, called the Trust Principle, is weaker than the standard Principal Principle but stronger than the New Principle. As I show, a rational agent will obey this principle if and only if she expects chance to be more accurate than she is on every good way of measuring accuracy. The Trust Principle requires chance to be a good guide to the world, permits modest chances, tells us how to listen to chance even when the chances are modest, and entails but is not entailed by the New Principle. I conclude by discussing the Trust Principle in the context of various metaphysical views of chance.

1 INTRODUCTION

Objective chance presents two types of problems. The first is metaphysical. We need to explain why some probability function or other is worthy of the honorific ‘chance’ at a particular world. The second is epistemological. Chance should guide our subjective degrees of belief, although how and why is a matter of contention. These questions are not separable. What chance is informs how we should let it guide us.

This paper focuses especially on the second type of question. I advocate a principle called the TRUST PRINCIPLE, that is weaker than the traditional Principal Principle [Lewis, 1980].¹ Very roughly: the Principal Principle tells you to defer completely to chance under normal circumstances. If you learn the chance of Heads is exactly 80%, then you should be 80% confident in Heads. The Trust Principle says that if you learn the chance of Heads is *at least* 80%, you should be *at least* 80% confident in Heads.

Why should we obey the TRUST PRINCIPLE? The crux of my argument rests on a new result that connects *accuracy* (how close a probability function comes to truth values) with *deference* (how some probabilities listen to other probabilities). The result entails that you will trust chance *if and only if* you always expect chance to be at least as accurate as you are when you have the same evidence.

I claim any good metaphysical account of chance will require rational agents to expect chance to be at least as accurate they are. Thus, the Trust Principle can serve either to constrain a metaphysical account of chance (such as Lewis’s Best Systems

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¹ Kevin Dorst [2019b] developed a similar principle in the case of rational credences, which inspired this work. The TRUST PRINCIPLE advocated here differs both in motivation and substance from K. Dorst’s principle, but I retain the terminology.

Account) or to provide a test of adequacy for that account. If the metaphysical account can't justify the Trust Principle, then it fails as an account of chance.

As we'll see, the TRUST PRINCIPLE has a number of attractive features. It:

- (i) Requires us to expect chances to be a good guide to the world,
- (ii) Permits but does not require modesty. That is, it allows the chance function to be uncertain that it is in fact the chance function,
- (iii) Places an upper bound on how modest chance functions can be,
- (iv) Tells us how we should listen to chance even when the chances are modest,
- (v) Entails the New Principle [Hall, 1994, Thau, 1994], and
- (vi) Entails the Principal Principle when chances are immodest.

Here's the plan. §2 discusses the notion of accuracy. §3 presents Lewis's views on chance and laws, along with the motivations for the Principal Principle and the New Principle. §4 argues for the TRUST PRINCIPLE and gives an informal version of the main result that connects accuracy and deference. §5 briefly applies the Trust Principle and accuracy considerations to various metaphysical accounts of chance. §6 wraps up. The Appendix provides a proof and general statement of the main result.

2 ACCURACY

The higher your credence in truths and the lower your credence in falsehoods the more accurate you are. This simple concept has spawned a movement in modern epistemology that invokes accuracy to justify a variety of core epistemic norms. Accuracy considerations underwrite arguments for probabilism, conditionalization, and even the Principal Principle.²

Despite its informal gloss, however, accuracy is a tricky notion to formalize. The *sine qua non* of any good measure is:

TRUTH DIRECTEDNESS Suppose b and c are probability functions defined over some algebra \mathcal{F} , w is a possible world, and X is a proposition. Let $w(X) = 1$ ($= 0$) if X is true (false) at w . If

- (i) $|b(X) - w(X)| \leq |c(X) - w(X)|$ for all $X \in \mathcal{F}$, and
- (ii) $|b(X) - w(X)| < |c(X) - w(X)|$ for some $X \in \mathcal{F}$,

then b is more accurate at w than c is.

In other words, if b 's credences are uniformly at least as close as c 's to truth-values at a given world, and for some proposition b 's credences are strictly closer, then b is more accurate at that world.

Accuracy theorists require a bit more, however, for a measure of accuracy to do the work they want. I too will assume these additional constraints on measures of accuracy, but since they are all standard in the literature, I won't say much to justify them here.

The most important such constraint is:

PROPRIETY Let Acc be a measure of accuracy. Acc is *proper* if for any distinct probability functions b and c , $E_b(\text{Acc}(b)) > E_b(\text{Acc}(c))$, where $E_b(\text{Acc}(\cdot))$ denotes the expected value of Acc according to b .

Propriety requires every probability function to estimate that it is more accurate than any other probability function. Without propriety certain probability functions are automatically rationally verboten no matter what evidence one possesses. If b

² The literature is large, but for probabilism, see [Joyce, 1998, 2009, Pettigrew, 2016]; for conditionalization, see [Greaves and Wallace, 2006], and for the Principal Principle, see [Pettigrew, 2012, 2013, 2016].

expects b' to be more accurate, then any agent whose credences are represented by b should switch to b' (assuming all she cares about is accuracy).³

The second requirement is slightly more involved. We can measure the accuracy not just of an entire probability function but also of individual credences. For instance, a credence of .7 in a truth is more accurate than a credence of .6 in a truth. I'll use $a^X(x, i)$ to denote the accuracy of a credence of x in proposition X whose truth-value is i (either 1 or 0). We then have:

ADDITIVITY $\text{Acc}(b, w) = \sum_{X \in \mathcal{F}} a^X(b(X), w(X))$ for some local measures of accuracy a^X .

Additivity requires that an agent's total level of accuracy is nothing over and above the accuracy of each of her individual credences.⁴

This definition is extremely permissive, in that it allows for different measures of accuracy for different individual propositions. I.e., $a^X(x, i)$ may not be the same as $a^Y(x, i)$. For instance, a standard measure Acc could score propositions about the weather with the Brier Score and propositions about politics with the Log score (defined below). So long as the total score Acc is simply the sum of the individual scores, it counts as additive.

Finally, there are continuity and limit requirements that are explored in the Appendix (Definition (A.1), conditions (iv) and (v)), but that I won't bother articulating precisely here.

I will refer to any measure of accuracy that is *additive, proper, truth-directed, and that meets the continuity and limit requirements as standard*. To make matters concrete, the following two measures are standard:

$$\text{BS}(b, w) := 1 - \sum_{X \in \mathcal{F}} (b(X) - w(X))^2 \quad (\text{Brier})$$

$$\text{Ln}(b, w) := \sum_{X \in \mathcal{F}} \ln(|(1 - w(X)) - b(X)|) \quad (\text{Log})$$

The Brier Score looks at the squared Euclidean distance between your credences and the truth. The Log Score adds the logarithm of your credence if the proposition is true, and the logarithm of $(1 - \text{your credence})$ if it's false. Both give you better scores as your credences come closer to truth values, but they encapsulate different notions of 'proximity to truth'. Indeed, there are infinitely many different standard measures.

2.1 Accuracy and Epistemic Superiority

With the notion of *standard measure* in hand, we can now formulate some notions of epistemic superiority. One natural notion is that Alice is an epistemic superior of Bob's if Alice is more accurate according to some particular standard measure. For example, if Alice and Bob are weather-forecasters, we might say that Alice is Bob's superior if she got a better Brier Score on her forecasts over the last year.

This notion of superiority is fine as far as it goes, but different standard measures will disagree about who's more accurate. Consider, for instance, the following example adapted from [Levinstein \[2019\]](#). Suppose there are four weighted balls in an urn (A, B, C, and D). Imagine that Alice and Bob's credences are as in Table 1.

Suppose that as a matter of fact A is drawn. It's unclear who should count as more accurate overall. On the one hand, Bob has a higher credence in the true proposition A, but it's still quite low. Bob also has a very high credence in the false proposition D.

The Log and Brier scores disagree here. The Brier score counts Alice as more accurate than Bob, but the Log score counts Bob as more accurate than Alice. There's

³ There is an extensive literature on propriety. See, for instance, [[Joyce, 1998, 2009, Pettigrew, 2016](#)].

⁴ For a defense, see [[Pettigrew, 2016](#), Ch. 4.1].

	A	B	C	D
Alice	.005	.275	.230	.490
Bob	.033	.127	.137	.703

Table 1: Alice and Bob’s credences that any particular ball will be drawn.

no answer to the question of which one of them is more accurate than the other unless we specify which measure we’re talking about.

There are, however, stronger notions of epistemic superiority. Instead of identifying epistemic superiority with being better on some particular standard measure, we can identify it with superiority on *all* standard measures. Alice and Bob will be incomparable in this case, since neither is strictly better than the other on all measures. However, imagine that Carol has credence .5 in A, .1 in B, .1 in C, and .3 in D. Carol is more accurate than Alice and Bob in each proposition, and (by Truth-Directedness), she’s therefore more accurate overall.

Actual superiority on every rule is very strong. A more useful criterion is the following:

ABSOLUTE SUPERIORITY Bob takes Alice to be *an absolute superior* with respect to some proposition X just in case he *expects that Alice will have a more accurate credence than he does in X according to every standard measure.*

Here’s the idea. Suppose I look out the window and form credence .3 that it will rain later today. I haven’t yet checked what the weather-forecaster says about rain. However, I expect that the weather-forecaster will be more accurate than I am. Indeed, it doesn’t matter what standard measure I use. I will still expect the forecaster to beat me.

Now, this thought may at first seem to conflict with **PROPRIETY**. Propriety says that on any standard measure, every probability function expects itself to be more accurate than any other probability function. The key here is that, in the definition of **PROPRIETY**, the other probabilities are named *de re*. However, the definition does not entail that you expect to be more accurate than any other probability *de dicto*.

To see the difference, suppose as before I have credence .3 that it will rain. That entails I must expect .3 to be more accurate than credence 1 and more accurate than credence 0. However, I *do not* expect .3 to be more accurate than the Omniscient Credence Function, which inevitably will assign credence 1 or credence 0 to the proposition that it will rain. Likewise, I can expect that the weather-forecaster’s credence (whatever it may be) will be more accurate than mine.⁵

Since this notion of absolute superiority is based on *expectations*, Bob can take Alice to be an absolute superior even when her credence turns out to be less accurate than his. That makes sense. When I look out the window and form credence .3 that it will rain, I expect that the weather forecaster will do better. It may turn out, of course, that she has credence .8 and that it does not rain. In this instance, I ended up more accurate than she did, but I still considered her my superior.

Furthermore, this notion of superiority is useful when thinking about revising one’s beliefs based on the views of others. How Bob changes his opinion based on

⁵ From a formal point of view, the idea works as follows. Suppose Bob is unaware of what Alice’s credence in X is. Alice’s credence, which we denote \mathcal{A} , is then a random variable. For simplicity, assume that \mathcal{A} is sure to take one of the values in $Q = \{q_1, \dots, q_n\}$. Then Bob calculates the expected score Alice will get for some standard measure as follows:

$$E_b(\text{Acc}(\mathcal{A})) = \sum_{q_i \in Q} b(\mathcal{A} = q_i) E_{b(\cdot | \mathcal{A} = q_i)}(\text{Acc}(q_i))$$

So, to calculate Alice’s expected score, Bob considers how he would update his beliefs upon learning that Alice’s credence took on some particular value and then calculates her expected score from the point of view of those updated beliefs. He then averages these conditional beliefs based on how likely he thinks it is that Alice will have those particular credences.

what he learns Alice thinks depends on his expectations about Alice, not on how accurate Alice actually turns out to be.

One fact that follows immediately from our [MAIN RESULT](#) below is that if Bob totally defers to Alice, he takes her to be an absolute superior. That is, if for all x , $c(X|A = x) = x$, then he expects Alice to be more accurate than he is according to every standard rule.⁶ However, as we'll also see below, Bob need not be quite so deferential to see Alice as his absolute superior.⁷

3 CHANCE AND DEFERENCE

We now turn to the discussion of chance. As mentioned above, chance plays a dual role. On the one hand, it is a type of objective probability, a feature of the world independent of what anybody thinks. On the other, it guides rational credence. For some probability function to count as the chance function at a world, it must successfully play both roles.

Of course, there are many different metaphysical views of what chance is. For the sake of simplicity, I will focus primarily on David Lewis's Best Systems Account, which is the most discussed account of what chance is in the literature. Below, I'll briefly consider alternative versions of the Best Systems Account as well. Ultimately, however, I'll argue the adequacy conditions that entail we should obey the Trust Principle are required of any good account, Best Systems or not.

3.1 Lewis on Chance

[Lewis \[1980\]](#) attempted to codify the right relationship between chance and credence with his famous Principal Principle (PP). There are now a variety of versions of the Principal Principle not all of which are equivalent. I will give a significantly simplified version here:

PRINCIPAL PRINCIPLE Let c be a rational credence function with total evidence E and let X be any proposition. Then

$$c(X|Ch^E(X) = x) = x,$$

assuming $c(Ch^E(X) = x) \neq 0$.

Some important notes. First, Ch refers *de dicto* to the chance function, whatever it is, at the start of the universe. Second, $Ch^E(X)$ abbreviates $Ch(X|E)$. Third, although the chance function evolves over time, we suppress any time indices since Ch is fed c 's total evidence (unlike in Lewis's version of the Principal Principle). Fourth, again since Ch is brought up to speed on c 's evidence, we do not need to worry about admissibility.⁸ Fifth, from here on out, I will ignore caveats such as "assuming $c(Ch^E(X) = x) \neq 0$." This formulation, though different from Lewis's, will serve as our official version of the Principal Principle.

Note that this version of the [PRINCIPAL PRINCIPLE](#) leaves out much of metaphysical and epistemological import to Lewis. Lewis wanted chance at a given time to screen off nearly all other types of evidence that an agent could acquire, which is why admissibility and the time-evolution of chance have been such vexing topics over the years. Nonetheless, these issues are orthogonal to the main points in

⁶ Except in the trivial case, that is, where he's certain $A = c(X)$.

⁷ Note that requiring $c(X|A = x) = x$ is very strong in general. I normally defer to my weather-forecaster, but if she were to say that she had credence 1 that it will snow in Phoenix on July 5, I would not myself be so sure.

⁸ My own favored view of inadmissibility comes from [Pettigrew \[2016\]](#). E is admissible for Ch with respect to some proposition X if and only if $Ch(X|E) = Ch(X)$. I.e., chance itself considers E independent of X . For a helpful discussion of admissibility, see [Meacham \[2010\]](#).

this essay, so I invoke this somewhat watered-down alternative to Lewis's official formulation.

Although the **PRINCIPAL PRINCIPLE** seems on the right track, there are a number of questions and problems that remain. The **PRINCIPAL PRINCIPLE** is so-called because Lewis thought it "captures all we know about chance" (p. 86). More broadly, however, we might say that what we know about chance is that it should guide our credences in some way or other. Still, we might wonder:

QUESTION 1 Why should we listen to chance?

One answer is flat-footed: because chance plays the chance-role. But the real answer to this question depends on the underlying metaphysical account of chance. What is it about the chance function's connection to the world that makes it such that rational agents ought to listen to it?

Suppose a metaphysician tells you that chance *just is* whatever her friend Belinda thinks. Belinda has no special epistemic insight, and we have no reason to think that she's especially accurate or somehow tuned into the underlying laws of nature. If chance just is whatever Belinda thinks, then we have no reason to listen to chance. Put differently, this Belinda-based view of chance is inadequate because Belinda's credences aren't suited to play the epistemic chance role.

We saw above that if Bob is willing to defer to Alice no matter what she thinks, then he expects Alice to be more accurate than he is. I.e., if $b(X|A = x) = x$ for all x , then Bob treats Alice as an absolute superior. So, if it's rationally required to defer to chance with respect to any proposition, then it's rationally required to expect chance to be more accurate than you are.

Therefore, if the **PRINCIPAL PRINCIPLE** is right, the chances must be expected to have some strong connection to the actual truth-values. For if not, then there's no reason that it's rationally obligatory to take chance to be an absolute superior.

There is thus a burden on the metaphysician who wishes to vindicate a principle like **PRINCIPAL PRINCIPLE** to explain why her account of what chance is will require rational agents to expect chance to be more accurate than they are. Over a series of papers, especially his [1980, 1994], Lewis developed a view of laws of nature and chance. This view, at least at first, seems to provide an answer to **QUESTION 1**.

3.2 Lewis's Best Systems Account

According to Lewis, chances are part of the laws of nature, and the laws themselves are the best way of codifying regularities of the world.⁹

Let's first start with the deterministic case. On Lewis's 'Best Systems' view, deterministic laws of nature just are true generalizations about categorical facts that maximize the virtues of simplicity and strength. Although the exact meaning of these terms is unclear, examples are helpful. The claim that at least one thing exists is simple, but it isn't strong, as it does little to distinguish the actual world from many other alternatives. The conjunction of all true facts is strong, but it is not simple. Some truths, such as Maxwell's equations are both simple and strong.¹⁰

Call any set of true statements about the world a 'lawbook'. *Strength* is a measure of how much of modal space the lawbook rules out. *Simplicity* is a measure of the syntactic length of the lawbook.¹¹ The lawbooks compete to be the laws of nature at a world based on how well they realize the Lewisian virtues of simplicity and strength. The winning lawbook, i.e., the actual laws of nature, is the collection of true generalizations that "strike the optimal balance" between these virtues.

⁹ My exposition of Lewis closely follows Weatherson [2016]. I omit a number of important metaphysical details that are not relevant to the main purpose of this paper, however.

¹⁰ Maxwell's equations are not strictly speaking true, but that doesn't really matter for our purposes.

¹¹ Lewis [1983] recognizes that this simplicity criterion requires a privileged language. Otherwise, we could create an unnatural language with the predicate K that is equivalent to the conjunction of all truths in the actual world. I.e., Kw is true only of the actual world.

The motivation for introducing indeterminacy (and chance) is that it allows for much simpler laws. For illustration, imagine a coin is flipped a thousand times, landing Heads roughly half the time and Tails roughly half the time. Although a full description of exactly how the coin lands each time is strong, it is not simple. Much simpler is a chance function that says that assigns probability .5 to the coin landing Heads on any given flip and that renders the flips independent of one another.

However, unlike in the deterministic case, it's hard to see what could make chance claims *true*. While it's clear what it would take for Maxwell's equations to be true, it's less obvious what it would take for it to be true that there is a .5 chance of a coin landing Heads.

To fix this problem, Lewis introduces a third theoretical virtue called *fit*. On Lewis's official view, one probability function fits a world better than another just in case it renders that world more likely. Given the potential for indeterminacy, then, the actual laws just are the set of claims that strike the best balance between strength, simplicity, and fit.

Two things to note. First, neither strength nor simplicity is fully cashed out. For our purposes, that won't matter much. However, *fit* is clearly defined: the higher the probability assigned to the actual world, the better the fit. As stated, it sounds a lot like accuracy, though as we'll see shortly it's slightly different.

Second, if Lewis is right both about the three virtues that determine what the chances are *and* about the [PRINCIPAL PRINCIPLE](#), then that constrains what 'strike the optimal balance' can mean. For all that was said about the three virtues, an ill-fitting probability function can be so simple and strong that it still counts as the chance function.

However, if anything like the [PRINCIPAL PRINCIPLE](#) is correct, that can't be so. According to the [PRINCIPAL PRINCIPLE](#), rational agents are required to conform their credences to chance in a very strong way. And if you rationally expect that chance is ill-fitting, then there's no reason to conform your credence to it, since you'll only defer to a different probability function if you expect it will be at least as accurate as you are.

Therefore, even candidate laws that do extremely well on simplicity and strength are automatically disqualified if they assign highly inaccurate objective probabilities. Such probabilities cannot be the chances because they will not play the role of chance in the lives of rational epistemic agents.

3.3 Accuracy as Fitness

Before moving on, I'd like to propose a friendly amendment to Lewis's account. Instead of identifying fit with the probability assigned to the actual world, we should instead identify fit with accuracy.

The reason: if we only look at the probability assigned to the actual world, we ignore what chance says about important propositions true of more than one world. Since the [PRINCIPAL PRINCIPLE](#) applies to *all* propositions, we likewise need to judge candidate chance functions based on how well they do on *all* propositions, not only on the proposition true just at the actual world.

To see why, consider a very simple world where a coin is flipped ten times. According to the official *fit* criterion, any two probabilities that assign equal chance to the actual sequence of heads and tails are equally fit, since fitness is sensitive only to the probability assigned to the actual world.

For ease assume we have 4 Heads followed by 3 Tails followed by 1 Head and 2 Tails. Since fitness officially does not account for the probability assigned to any other sequence, probability functions get no credit for being confident that: the coin comes up heads exactly five times, the coin comes up heads between three and seven times, the coin will not always land tails, the coin comes up heads on the first flip, etc. These are all propositions true of more than one sequence, so they

are not directly relevant to a probability's fitness score. Thus, in general, rational agents have insufficient reason to trust probability functions about these propositions merely because they achieve good fitness scores.

Instead, we should score probabilities based on how they do over the entire space of propositions. They should get better scores for assigning high credence to truths and low credence to falsehoods. But that *just is* accuracy.¹²

Henceforth, I shall identify fitness with accuracy. Of course, since there are different measures of accuracy, some probability functions will turn out to be incomparable with one another in terms of fitness. However, as we'll discuss, this fact won't stand in the way of deference if we require chance to be sufficiently accurate. If you take chance to be an absolute superior—if you expect it to be more accurate than you on every standard measure—you'll defer to it. Before seeing how that works, however, we should understand where the **PRINCIPAL PRINCIPLE** conflicts with Lewis's metaphysical account of chance.

3.4 Modest Chance

Sadly, if Lewis's fit criterion is correct, then **PRINCIPAL PRINCIPLE** is false. To see why, consider this example adapted from Lewis [1994] and Weatherson [2016]:

DECAY Let A be the proposition that every tritium atom will decay within 12.32 years, which is tritium's actual half-life. A has chance $1/2^n$ of occurring, where n is the number of tritium atoms in the world. But if A occurred, the best system of the world would be different from how it actually is.

This case presents the problem of undermining futures. Suppose ρ is the actual chance function. That is, the claim that ' ρ is the chance function' is true at the actual world, where ρ is specified *de re*. ρ assigns A a positive probability. However, if A were to occur, ρ wouldn't be the chance function since some alternative function would fit the world better. Or, put differently, the half-life of tritium wouldn't be 12.32 years, but instead it would be some lower number.

So, since ρ can't be the chance function if A were to occur, and ρ thinks A has probability greater than 0, ρ is not certain that it is the chance function. Letting Ch_ρ stand for the proposition that ρ is the chance function, we have $\rho(Ch_\rho) < 1$. In other words, ρ is *modest*.

Unfortunately, the potential for undermining futures renders the **PRINCIPAL PRINCIPLE** inconsistent. By the **PRINCIPAL PRINCIPLE**, $c(Ch_\rho | Ch_\rho) = \rho(Ch_\rho)$. The laws of probability require $c(Ch_\rho | Ch_\rho) = 1$. So, $\rho(Ch_\rho)$ must be 1 if the **PRINCIPAL PRINCIPLE** is true.¹³

Hall [1994] and Thau [1994] independently came up with a fix.¹⁴ The problem with the **PRINCIPAL PRINCIPLE** is that the agent supposes ρ is the chance function, but ρ is not brought up to speed on the fact that it is the chance function. So, they proposed:

NEW PRINCIPLE Let c be a rational credence function with total evidence E . Then for any proposition X and any possible chance function ρ ,

$$c(X | Ch_\rho) = \rho(x | E, Ch_\rho).$$

For contrast, note that according to the **PRINCIPAL PRINCIPLE**, if you learn ρ is the chance function, then you defer to ρ conditioned on your evidence. I.e., $c(X | Ch_\rho) = \rho(X | E)$.

¹² See Hicks [2017], discussed below, for another view that identifies fitness with accuracy.

¹³ This technically requires a very slightly stronger version of the **PRINCIPAL PRINCIPLE**.

¹⁴ For a separate fix, see Ismael [2008] along with objections by Pettigrew [2015]. Roberts [2013] also suggests a general principle that constrains comparative confidence and that turns out to be weaker than the **PRINCIPAL PRINCIPLE**.

With the **NEW PRINCIPLE**, you don't listen to the chances themselves.¹⁵ Instead, you listen to the chances conditioned on the claim that they're the chances along with whatever other evidence you have. In our **DECAY** example above, $\rho(A|Ch_\rho) = 0$, since if ρ knows it's the chance function then it knows that A must be false.

To avoid excessive verbiage, I'll refer to the chances conditioned on the fact that they're the chances as *the chances*⁺.

Despite some qualms, Lewis [1994] ended up embracing the **NEW PRINCIPLE**. Instead of conforming your credences to the chances, you should conform them to the chances⁺. With the chances⁺, we end up with a consistent principle for modest chances instead of the inconsistent **PRINCIPAL PRINCIPLE**.

Let's take stock of where we are. First, we introduced a 'fitness' criterion for chance, which requires chances to assign high probability to the actual world modulo simplicity and strength. We modified this criterion to an accuracy criterion, which underwrites any rational requirements for conforming credences to chance. However, the accuracy requirement also led to undermining futures, which renders the **PRINCIPAL PRINCIPLE** inconsistent. We then introduced the **NEW PRINCIPLE**, which is consistent and also can guide rational credence.

3.5 Modest Chance and Credence

A question remains, however. If we adopt the **NEW PRINCIPLE**, then we're left without any principle for how the regular chances, not the chances⁺, can guide our credences. If the chances cease to have any epistemic role, then why care about them? Even if we adopt the **NEW PRINCIPLE**, we need to know:

QUESTION 2 If the chances are modest, how should they guide our credences?

Unlike **QUESTION 1**, there's currently no proposed answer to **QUESTION 2** in the literature that I am aware of.

One answer is that, as far as epistemology goes, we can forget about the chances and just focus on the chances⁺. However, it's important to have some credence-chance norm even when the chances are modest for the following reasons.¹⁶

First, actual scientific and epistemic practice primarily pay heed to the normal chances. After all, we normally calculate the chance, not the chance⁺, of a particle decaying before its half-life.

Second, we have better epistemic access to the normal chances. To see why, note that we can learn about the chance of individual propositions more easily than we can learn the chance⁺ of those propositions. For example, we can determine the half-life of tritium or know the chance of a coin landing heads is .5. However, to know the chance⁺ of a coin landing heads, we need to know what the chance function would say about heads conditional on the fact that it's the chance function. It's hard to know what would happen to the normal chance function conditioned on this information.

Third, the chances⁺ have odd features. For instance, they come apart from any faithfulness to causal structure. To see why, observe that in our **DECAY** example, the tritium atoms are causally independent from one another. The chances treat them as such and calculate the probability of any n of them decaying before their half-life as $1/2^n$. However, the chances⁺ treat them dependently, since the chance⁺ of one of them decaying before its half-life is $1/2$, but the chance⁺ of all of them decaying before their half-lives is 0. It is especially inelegant to say that the chances play the important metaphysical role in laws, but the chances⁺ are what matters for credence. After all, on Lewis's view, chances earn their name by exhibiting a

¹⁵ For technical reasons discovered by Grant Reaber and explained in [Pettigrew and Titelbaum, 2014, p. 6], ρ should be a candidate *initial* chance function, i.e., a chance function that has not yet conditioned on any evidence.

¹⁶ For an argument that the **NEW PRINCIPLE** is unmotivated, see [Briggs, 2009].

high degree of fitness. This criterion is in place primarily for epistemic purposes, so chances should play some important epistemic role on their own.¹⁷

Fourth, even if we defer to the chances⁺, the **NEW PRINCIPLE** leaves our views of the chances almost entirely unconstrained. We need a unified account that applies to both the chances and the chances⁺.

We will return to Lewis's account and other versions of the Best Systems view below. Now, we examine more closely the connection between accuracy and deference.

4 ACCURACY AND DEFERENCE

So far, we've suggested that accuracy is what underwrites deference. The idea was that by building an accuracy-like criterion into the metaphysics of chance, we could motivate a deference principle such as the **PRINCIPAL PRINCIPLE** or the **NEW PRINCIPLE**.

As we'll now see, the connection between accuracy and deference is more subtle than one might think. And, as I'll argue, it will motivate a new deference principle that's weaker than the **PRINCIPAL PRINCIPLE** but stronger than the **NEW PRINCIPLE**.

Historically, philosophers have generally been interested in *expert* principles. An expert principle tells you to defer entirely to some other probability function. The **PRINCIPAL PRINCIPLE** and **NEW PRINCIPLE** are expert principles because they say that if you learn the chance (chance⁺) of some proposition is exactly x , you should have credence exactly x .¹⁸

Kevin Dorst [2019b] recently noted another type of interesting deference principle that's weaker than an expert principle.¹⁹ Following his terminology, we first introduce the following principle:

SIMPLE TRUST Let c be a rational credence function with total evidence E . Then

$$c(X | Ch^E(X) \geq x) \geq x$$

for all propositions X .

First, note that **SIMPLE TRUST** is symmetric. If $c(X | Ch^E(X) \geq x) \geq x$ for all x , then $c(X | Ch^E(X) \leq x) \leq x$ for all x as well, since if X is a proposition, so is $\neg X$, and if you know $Ch^E(X) \geq x$, you know $Ch^E(\neg X) \leq 1 - x$. Likewise, if there are only finitely many possible chance functions, **SIMPLE TRUST** entails that $c(X | Ch^E(X) > x) > x$ and $c(X | Ch^E(X) < x) < x$.

Second, note that **SIMPLE TRUST** does not entail any expert principle. It tells you what to do if you learn the chance of some proposition is at least .7, and it tells you what to do if you learn it is no more than .8, but it doesn't tell you what to do if you learn that it is at least .7 and *at most* .8.²⁰

Why care about **SIMPLE TRUST**? The answer comes from our main result. We state a rough version here and a more general and precise version the Appendix:²¹

¹⁷ For more on this point, see Arntzenius and Hall [2003].

¹⁸ Other expert principles include the principle of Reflection [van Fraassen, 1984] and Rational Reflection [Christensen, 2010].

¹⁹ The formulation here differs in some important technical ways from Dorst's formulation, but it's close enough to retain the terminology.

²⁰ This follows from the fact that from $c(X | E) = x$ and $c(X | F) = y$, we can't infer what $c(X | E \wedge F)$ is for arbitrary propositions X , E , and F .

²¹ There is one important caveat to flag here. The result assumes that $Ch(X)$ can take only one of finitely many values. Philosophically, we can justify this if we assume there are only finitely many worlds, since each world has only one chance function. How to generalize the result is a subtle and as yet undetermined question.

MAIN RESULT Let c be a probability function with total evidence E . Then c simply trusts chance if and only if

$$E_c(\text{Acc}(Ch^E)) \geq E_c(\text{Acc}(c))$$

for all standard measures of accuracy Acc with equality only when $c(Ch_c) = 1$.

In other words, you simply trust chance just in case you expect chance to be more accurate than you are on *every standard measure of accuracy* (unless you're already certain your credence function is the chance function). Since standard measures of accuracy obey **ADDITIVITY**, simply trusting chance requires you to think chance is at least as accurate as you are on every single proposition.

SIMPLE TRUST is also consistent with modesty. To see why, suppose there are only two worlds: w_1 and w_2 . Let p be the chance function at w_1 and q be the chance function at w_2 . Suppose $p(w_1) = .7$, and $q(w_1) = .3$, so both p and q are modest. For any c such that $0 < c(w_1) < 1$, $c(w_1 | Ch(w_1) \geq x)$ will be 1 when $x > .3$ and will be $c(w_1)$ otherwise. The same goes for w_2 *mutatis mutandis*. If, in addition, $.3 \leq c(w_1) \leq .7$, then c simply trusts chance.

SIMPLE TRUST is weaker than the Principal Principle, but simply trusting the chances comes with strong accuracy commitments. Indeed, since expecting the chances to be more accurate is necessary and sufficient for Simple Trust, this principle appears at first to be the best we can get from an accuracy criterion alone. However, as we'll now see, we can do a bit better.

4.1 The Trust Principle

We can strengthen **SIMPLE TRUST** to:²²

TRUST PRINCIPLE Let c be a rational credence function, and let E entail c 's total evidence. Then for any proposition X ,

$$c^E(X | Ch^E(X) \geq x) \geq x.$$

The difference between the **TRUST PRINCIPLE** and **SIMPLE TRUST** is just that the former requires agents to have credence at least x when the chances have credence at least x *no matter what information the agent might learn, so long as chance is fed at least that same information*. That's why for **TRUST PRINCIPLE** we consider any proposition that entails c 's total evidence, whereas for **SIMPLE TRUST**, we only consider the evidence c actually has.

This distinction doesn't matter for the formulations of the **PRINCIPAL PRINCIPLE** and **NEW PRINCIPLE** because both of those are closed under conditionalization. If you start out your life obeying either the **PRINCIPAL PRINCIPLE** or **NEW PRINCIPLE**, you'll continue to obey them as you acquire more information (so long as you always update by conditionalization).

SIMPLE TRUST, however, is not closed under conditionalization. It's possible to simply trust chance at first but to stop simply trusting chance after learning new information even if both you and chance update by conditionalization.

There are two reasons to move to the **TRUST PRINCIPLE** from **SIMPLE TRUST**. First, we want chance to continue to be a guide to the world (and be highly accurate, at least in expectation) as chance itself learns new things over time. Chance should not start out reliable at the time of the big bang but then lose its guidance value as time goes on.

Second, this principle is the strongest we'll be able to motivate from accuracy considerations alone (unless we identify accuracy with truth). By the Main Result, the **TRUST PRINCIPLE** entails that *all* rational agents are *required* to expect chance to be at least as accurate as they are on *every* standard measure of accuracy with

²² Again, this terminology follows K. Dorst [2019b] despite some substantive differences.

respect to *every* proposition *whenever* chance knows at least as much as they do. That is, as a corollary of the [MAIN RESULT](#), you trust chance just in case you'll take chance to be an absolute superior no matter what you both might learn.

4.2 An Adequacy Condition

Whether chance can be trusted depends on the underlying metaphysics. Although we've focused on Lewis, any good account of chance should require that it's worth listening to. For those inclined toward Best Systems Accounts, we can bake in a strong connection between chance and the world. That is, we can require that chance is, by its nature, accurate since accuracy is one of the very virtues that determines which probability function is the chance function at a world. From such an accuracy-focused metaphysics, we can then derive a deference principle.

One way to specify exactly how accurate chance needs to be is to endorse the following adequacy condition on metaphysical accounts of chance:

ACCURACY ADEQUACY A metaphysical account of chance is *adequate* only if rational agents with any body of total evidence E should expect the chance function conditioned on E to be at least as accurate as they are on any standard measure of accuracy.

In other words, rational agents should expect that chance is an *absolute* epistemic superior to them so long as chance has at least the same information that they do: on any decent measure of accuracy, chance is expected to beat you. Given the [MAIN RESULT](#), we have that iff chance meets [ACCURACY ADEQUACY](#), then rational agents will obey the [TRUST PRINCIPLE](#).

One way to motivate [ACCURACY ADEQUACY](#) for Best Systems theorists is as follows: Chance should be both a *database* and *analyst* expert in the terminology of [Hall \[2004\]](#). You treat someone as an analyst expert if you think she's at least as good as you are at handling evidence. For example, suppose you weren't sure what to make of your most recent X-rays. You have no experience reading X-rays and can't tell how likely it is you have a small fracture. Your doctor can look at the same information and form a much better credence than you can. She is an analyst expert. A database expert, on the other hand, is someone you trust because she knows more than you do. If you don't know anything about baseball, you can ask your friend who won the last World Series. He is a database expert.

Chance, on a Best Systems View, can play both roles. Chance is fed at least the same information you have, so it's at least as good as you are in database terms. However, even if you're rational, it will be better at analyzing evidence than you are. The probability function that counts as chance is determined based on how it fits the entire history (past, present, and future) of the world. High levels of accuracy can be hard coded in. Rationally responding to your evidence, however, does not entail you will in fact be accurate, but instead only that you handle your evidence in accord with whatever epistemic norms are correct. Although you may be a good analyst, you are still at a disadvantage.

4.3 Deference Adequacy

The [MAIN RESULT](#) is biconditional. So, we could start from the other direction. In lieu of [ACCURACY ADEQUACY](#), we could insist upon:

DEFERENCE ADEQUACY A metaphysical account of chance is *adequate* only if rational agents should defer to the chances in an appropriate way.

That is, we could begin with the idea that chance must be able to guide our credences in the right way instead of beginning with the idea that chances should be sufficiently accurate in expectation. From this starting point, we can still make a

case that the **TRUST PRINCIPLE** is the minimum standard of adequacy for chance to play the chance role.

To motivate this starting point, we can begin from a similar point to Lewis's own. As noted earlier, Lewis called his deference principle the **PRINCIPAL PRINCIPLE** because he thought it captured all we knew about chance. Although the particulars turned out wrong, we can still say that the primary thing we know about chance is that we need to defer to it in some way such that it can guide our credences. From there, we can figure out how exactly it should guide our credences and then determine what metaphysical constraints that imposes on chance.

What makes **TRUST PRINCIPLE** attractive as a deference principle independently of accuracy considerations?²³

First, the **TRUST PRINCIPLE** entails (but is not entailed by) the **NEW PRINCIPLE**.²⁴ So, we recover the most common accepted principle for modest chances, but we gain another principle that constrains deference to normal chance, not just the chances⁺. We thus answer **QUESTION 2**. Modest chances still have a crucial role in guiding our credences.

Second, if the chances are immodest, the **TRUST PRINCIPLE** is equivalent to the **PRINCIPAL PRINCIPLE** (since the immodest case is just a special case of the **NEW PRINCIPLE**). Unlike the **PRINCIPAL PRINCIPLE**, however, the **TRUST PRINCIPLE** is compatible with *modesty*. If chances are modest, we presumably want a strong deference principle that is consistent. The **TRUST PRINCIPLE** plays that role.

Third, the requirement that metaphysical accounts vindicate **TRUST PRINCIPLE** places important but well-motivated constraints on those metaphysical accounts. For example, if ρ is the chance function at w and ρ' is the chance function at w' , $\rho(w) \geq \rho'(w)$. That is, the chance function at each world assigns that world at least as high a chance as the chance function at any other world does. To see why, suppose $.5 = \rho'(w) > \rho(w) = .4$. If you then learned that the chance of w was at least $.5$, you'd know that w is not the actual world since your credence in w should go to 0. Learning that the chance is sufficiently high should never lower your credence.

Another way to look at this constraint is as a limit on how modest chance functions can be. ρ can be unsure that it is the chance function. However, ρ has to be at least as confident that ρ is the chance function as any alternative candidate ρ' is that ρ is the chance function.²⁵ Thus, the **TRUST PRINCIPLE** places constraints on which functions can jointly be the chances at which worlds.

So, we have two routes to **TRUST PRINCIPLE** given the main result. Either we can start with some sort of Lewisian fit criterion and design the metaphysical account of chance so that it's rationally required to expect that chance will beat you, *or* we can start with **DEFERENCE ADEQUACY** and then argue that it leads to the **TRUST PRINCIPLE**.

Either way, we specify how even modest chances guide credences and recover the **PRINCIPAL PRINCIPLE** when chances are immodest and the **NEW PRINCIPLE** when they're modest.

23 See K. Dorst [2019b] for more on a principle very similar to the **TRUST PRINCIPLE**. One important difference to keep in mind is that K. Dorst is interested in rational but modest credence functions that all trust one another, whereas we're here considering rational functions that trust chance (even though chance doesn't trust them).

24 To see why the **TRUST PRINCIPLE** entails the **NEW PRINCIPLE**, consider $E = Ch_\rho$ for some ρ . If c trusts chance, then $c^E(X | Ch^E(X) \geq x) \geq x$ and $c^E(X | Ch^E(X) \leq y) \leq y$ when defined. But since E entails the value of $Ch^E(X)$, we have $c(X | E) = c^E(X | Ch^E(X) \geq x) = c^E(X | Ch^E(X) \leq y)$. So, $c(X | E) = c(X | Ch^E(X) = x)$ as the **NEW PRINCIPLE** requires. See Fact 5.3 of K. Dorst [2019b].

To see that the **NEW PRINCIPLE** does not entail the **TRUST PRINCIPLE**, suppose there are only two worlds: w and w' . Let ρ_w and $\rho_{w'}$ be the chance functions at w and w' . Suppose $\rho_w(w) = .4$ but $\rho_{w'}(w) = .6$. Then ρ_w can't be trusted (see below in the main text). However, upon learning that ρ_w is the chance function, ρ_w will become certain that w is the actual world, as will any rational agent. Therefore, in this case, a rational agent could obey the **NEW PRINCIPLE** without satisfying **TRUST PRINCIPLE**.

25 See §§5-6 of K. Dorst [2019b].

5 TRUST AND METAPHYSICAL ACCOUNTS OF CHANCE

We now apply the above discussion to particular versions of the Best Systems Account to illustrate the connection between chance, accuracy, and the [TRUST PRINCIPLE](#).

5.1 Back to Lewis

In the case of Lewis, we've already amended the account to identify fit with accuracy.

However, Lewis's view is still unfortunately vague. We have been given little guidance as to what counts as 'striking the best balance' between accuracy, strength, and simplicity. The above discussion makes some headway. Whatever the best balance is, it must require enough accuracy that rational agents should always expect chance to be trustworthy.

Given all Lewis has said, this is surely possible. Perfect accuracy is guaranteed to do at least as well as rationality. So long as rational agents sometimes assign credences other than 1 or 0 to some propositions, chance need not be identified with truth.

However, it rules out some ways in which other Lewisian virtues could compensate for fit. Regardless of how well a lawbook scores on simplicity and strength, it shouldn't count as the laws of nature at a world if its candidate chance function is insufficiently accurate. Although this does not determine what exactly is meant by 'strike the optimal balance' it does entail that chance must meet a minimum bar for fitness before the other virtues can decide which candidate wins.

5.2 Hicks' and C. Dorst's BSA Accounts

Michael Hicks [2017, 2019] has recently proposed versions of a Best Systems Account different in detail from Lewis's. Hicks agrees that instead of Lewis's traditional *fit* criterion, we should use an accuracy criterion instead.²⁶

Hicks claims that chance is the function that maximizes accuracy subject to certain constraints. For example, in his [2019], he argues that the chance function is the maximally accurate probability function that is invariant given "local and intrinsic information." That is, in any situation where the local and intrinsic information is the same, the chances are the same.²⁷ This requirement prevents chance from being trivial: since chance obeys this invariance requirement, it won't be the same as truth. Hicks argues further that invariance given local and intrinsic information constrains rational belief as well: if two situations have the same local and intrinsic information, they are evidentially identical and require the same credence distribution.

Chris Dorst [2019a] emphasizes chance's predictive value. On his view, chance should specifically be designed so as to be maximally "predictively useful to creatures like us" (p. 898). That is, chances are, roughly, both discoverable and maximally useful for guiding our credences.

On both Chris Dorst's and Hicks' view, then, it's relatively easy to argue that we should obey the [TRUST PRINCIPLE](#). For Hicks, the constraints on chance other than accuracy are constraints that apply to rational credence as well. Chance operates under no more constraints than rational epistemic agents do and it maximizes accuracy under those constraints. Therefore, rational agents should expect chance to be at least as accurate as they themselves are and should in turn trust the chances.

²⁶ Hicks is right to require chances to maximize accuracy subject to constraints, though he is wrong in certain details. Hicks identifies accuracy with the Brier Score alone. Unfortunately, expecting chance to be more accurate on the Brier score is insufficient for the [NEW PRINCIPLE](#), which Hicks hopes to vindicate.

²⁷ For a related view arguing that chances are stable given intrinsic information, see Schaffer [2003].

For Chis Dorst, chance just is the system that maximizes predictive utility. So, by a similar argument, rational agents should trust the chances.

5.3 Alternatives to BSA

I've focused on versions of the Best Systems Account because BSA views are the most developed and discussed in the literature. Other views have, of course, been articulated. For instance, Tooley [1987] argues for a type of primitivist propensity theory. Imagine a simple world where a coin is flipped 1,000 times (and nothing else happens). If the coin lands heads 489 times, it does not seem to follow that the chance of heads is .489. Indeed, intuitively, the chance could be anything other than 0 or 1. Put differently, there could be two such worlds with the exact same patterns of heads and tails but with different chances.

Such primitivist views hold some intuitive appeal. However, it's opaque why, on these views, chance can play the necessary epistemic role. It's therefore especially incumbent upon metaphysicians endorsing non-BSA views to explain why we're rationally required to expect chance to be at least as accurate as we are or, alternatively, why we should defer to it. Without baking accuracy into the metaphysical account of chance (as BSA views do), the epistemic role for chance is harder to vindicate.²⁸

6 CONCLUSION

For chance to play the appropriate epistemological role, rational agents must expect it to be sufficiently accurate. How accurate? Enough that any rational agent should expect to be no more accurate chance on any good measure of accuracy on any proposition. How to ensure that chance plays that role is a task for metaphysical accounts of what chance is.

This accuracy requirement leads to the **TRUST PRINCIPLE**. The **TRUST PRINCIPLE** is stronger than the **NEW PRINCIPLE** but weaker than the **PRINCIPAL PRINCIPLE**. Indeed, it is the strongest principle we can get from accuracy considerations alone without identifying chance with truth. And, unlike the **NEW PRINCIPLE**, it tells us how chances should guide our credences even when they're modest.

A PROOF OF MAIN RESULT

In this appendix we state and prove a more careful and more general version of the **MAIN RESULT**.

First, we define:

Definition A.1 (Local Scoring Rule). A **local scoring rule** is a function $S : [0, 1] \times \{0, 1\} \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$ such that:

- (i) $S(x, 1) < S(y, 1)$ if and only if $x > y$, and $S(x, 0) < S(y, 0)$ if and only if $x > y$,
- (ii) $S(0, 0) = S(1, 1) = 0$
- (iii) For all $x, y \in [0, 1]$, $x \cdot S(x, 1) + (1 - x) \cdot S(x, 0) \leq x \cdot S(y, 1) + (1 - x) \cdot S(y, 0)$ with equality if and only if $x = y$
- (iv) $S(\cdot, 1)$ and $S(\cdot, 0)$ are continuous over $(0, 1)$, and
- (v) $S(i, j) = \lim_{t \rightarrow i} S(t, j)$ for $i, j = 0, 1$.

²⁸ For two non-BSA accounts that attempt to explain why we should defer to chance, see Lange [2017] and Elliott [2016]. See also Emery [2017] for more opposition to Humean accounts.

Note that local scoring rules measure the *inaccuracy* of a credence in a particular proposition. The higher (lower) your credence in a truth (falsehood), the less inaccurate you are. Item (i) corresponds to [TRUTH DIRECTEDNESS](#). Item (ii) is for technical convenience. Item (iii) corresponds to [PROPRIETY](#). Items (iv) and (v) are the continuity requirements alluded to in §2.

Since we require that standard accuracy measures obey [ADDITIVITY](#), we henceforth will only need to consider local scoring rules and credences in individual propositions. However, we will have to redefine the notion of trust:

Definition A.2 (Local Trust). Suppose p is a probability function, and A is a random variable specifying the (perhaps unknown) probability that some function assigns to proposition X . Then p (**locally**) **trusts** A just in case:

- $p(A \geq p(X)) > 0$
- $p(A \leq p(X)) > 0$
- $p(X|A \geq x) \geq x$ for all x (when defined)
- $p(X|A \leq x) \leq x$ for all x (when defined)

The reason for this definition is that if p is antecedently certain that $A < p(X)$, then $p(X|A \geq x)$ is either undefined or will be greater than x . Because only a single proposition is now involved, we also lose the distinction between trust and simple trust. For what follows, I'll simply refer to *local trust* as *trust* for convenience.

A.1 Notation

We hold p and X fixed as some agent's credence function and some proposition respectively. To save space, we define:

- $E_p(S(A))$: the expected score p assigns to A 's credence in X .
- $\pi := p(X)$
- \bar{X} for $\neg X$
- $p^{a_i}(X) := p(X|A = a_i)$
- $p(X, a_i) := p(X \wedge A = a_i)$, etc.
- We write $g_1(x)$ for $S(x, 1)$ and $g_0(x)$ for $S(x, 0)$.

One other major assumption: A will take only one of finitely-many values. Call the set of possible values

$$\mathcal{A} = \{a_0 = 0, a_1, \dots, a_{n-1}, a_n = 1\}$$

I.e., $p(A \in \mathcal{A}) = 1$. Setting $a_0 = 0$ and $a_1 = 1$ is purely for convenience, as $p(A = 0)$ and $p(A = 1)$ can both equal 0. The extent to which the main result will generalize to the infinite case is unclear.

A.2 Proof of Main Result

We make heavy use of the following representation of strictly proper scoring rules.

Theorem 1 ([Schervish \[1989\]](#)). $S = (g_1, g_0)$ is a local scoring rule if and only if there exists a measure $\lambda(dq)$ on $[0, 1]$ such that:

$$g_1(x) = \int_x^1 (1 - q) \lambda(dq)$$

$$g_0(x) = \int_0^x q \lambda(dq)$$

for all x , where λ gives positive measure to every interval $[a, b)$ where $b > a$.

From this, we prove the following key lemma to connect the Schervish representation with something resembling Trust.²⁹

Lemma 2. $E_p(S(A)) = \int_0^1 q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \lambda(dq)$

Proof. By the definition of expectation combined with Theorem 1, we have:

$$\begin{aligned} E(S(A)) &= \sum_{i=0}^n p(a_i) \left[p^{a_i}(X) \int_{a_i}^1 (1 - q) \lambda(dq) + p^{a_i}(\bar{X}) \int_0^{a_i} q \lambda(dq) \right] \\ &= \sum_{i=0}^n p(X, a_i) \int_{a_i}^1 (1 - q) \lambda(dq) + p(\bar{X}, a_i) \int_0^{a_i} q \lambda(dq) \end{aligned} \quad (1)$$

Consider the first term in Equation (1), i.e.:

$$\sum_{i=0}^n p(X, a_i) \int_{a_i}^1 (1 - q) \lambda(dq) \quad (2)$$

Note we are integrating the same function from a_n to 1 then from a_{n-1} to 1, etc. Since $a_n = 1$ and $g_1(1) = 0$, we can rewrite expression (2) as:

$$\sum_{i=0}^{n-1} p(X, A \leq a_i) \int_{a_i}^{a_{i+1}} (1 - q) \lambda(dq) \quad (3)$$

Since $p(X, A \leq a_i)$ is constant over $[a_i, a_{i+1})$, we re-write expression (3) as:

$$\sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} (1 - q) p(X, A \leq q) \lambda(dq) = \int_0^1 (1 - q) p(X, A \leq q) \lambda(dq) \quad (4)$$

The derivation for the second term of equation (1) is similar, but note we have a strict inequality in the first term of the lemma. Aside from the case of $a_n = 1$, note that $p(\bar{X}, A > a_i)$ is constant over $[a_i, a_{i+1})$ and that $g_0(0) = 0$. So, by similar reasoning to that above, we have:

$$\begin{aligned} \sum_{i=0}^n p(\bar{X}, a_i) \int_0^{a_i} q \lambda(dq) &= \sum_{i=1}^n p(\bar{X}, A > a_i) \int_{a_{i-1}}^{a_i} q \lambda(dq) \\ &= \int_0^1 p(\bar{X}, A > q) q \lambda(dq) \end{aligned} \quad (5)$$

Putting equations (4) and (5) together, we have the desired result. \square

To prove the main result, we need to connect the expected score p assigns A with the expected score p assigns itself. So,

Remark. By Theorem 1, $E_p(S(\pi)) = \pi \int_{\pi}^1 (1 - q) \lambda(dq) + (1 - \pi) \int_0^{\pi} q \lambda(dq)$.

We're now in a position to prove the main result:

Theorem 3. p locally trusts A if and only if $E_p(S(\pi)) \geq E_p(S(A))$ for every strictly proper scoring rule S . Moreover, the inequality is strict if $p(A = \pi) \neq 1$.

Proof. First we show the left-to-right direction. Suppose p trusts A . We appeal to the following fact:

Fact 4. If p trusts A , then for all $q \in [0, 1]$:

²⁹ After deriving this Lemma, I noticed that Schervish [1989] made some remarks that suggested it was true (see his Definition 3.1 on p. 1859), though he gave no proof.

1. $p(X, A \leq q) \leq q \cdot p(A \leq q)$
2. $p(X, A \leq q) \leq \pi - q \cdot p(A > q)$
3. $p(\bar{X}, A > q) \leq (1 - q)p(A > q)$
4. $p(\bar{X}, A > q) \leq (1 - \pi) - (1 - q)p(A \leq q)$

Moreover, if $p(A = \pi) \neq 1$, then each inequality is strict for *some* q .

Proof. To prove (1): If $p(A \leq q) = 0$, then clearly the inequality holds. If not, then $p(X|A \leq q) \leq q$, since p trusts A , and the definition of conditional probability ensures the result. Furthermore, if $p(A = \pi) \neq 1$, then there's some $q_0 < 1$ such that $p(A \leq q_0) > 0$. So, since \mathcal{A} is finite, and $q_0 \in [a_i, a_{i+1})$ for some $a_i \in \mathcal{A}$, there's some $\varepsilon > 0$ such that $q_0 + \varepsilon < a_{i+1}$, and $p(X, A < q_0 + \varepsilon) = p(X, A \leq q_0) < (q_0 + \varepsilon)p(A \leq q_0 + \varepsilon)$. The proof of (3) is similar to (1).

To prove (2): We know $p(X, A > q) \geq q \cdot p(A > q)$ since either both sides are $= 0$, or the definition of conditional probability immediately guarantees the inequality. $p(X) = \pi = p(X, A > q) + p(X, A \leq q)$, whence we obtain the desired result. Moreover, if $p(X, A > q) \neq 0$ and $p(A = \pi) \neq 1$, then the inequality is strict for some q . The proof of (4) is similar to the proof of (2). \square

By Lemma 2,

$$\begin{aligned}
 E_p(S(A)) &= \int_0^1 q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \lambda(dq) & (6) \\
 &= \int_0^\pi q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \\
 &\quad + \int_\pi^1 q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \\
 &\leq \int_0^\pi q(1 - \pi - (1 - q)p(A \leq q)) + (1 - q)q \cdot p(A \leq q) \lambda(dq) & (7) \\
 &\quad + \int_\pi^1 q((1 - q)p(A > q)) + (1 - q)(\pi - q \cdot p(A > q)) \lambda(dq) \\
 &= \int_0^\pi q(1 - \pi) \lambda(dq) + \int_\pi^1 (1 - q)\pi \lambda(dq) \\
 &= E_p(S(\pi))
 \end{aligned}$$

Line (7) follows from Fact 4. The inequality is strict if $p(A = \pi) \neq 1$. This completes the proof of the left-to-right-direction.

To show the right-to-left direction, we prove the contrapositive. Suppose p does not trust A . Then either (i) there exists $a_i \in \mathcal{A}$ such that $p(X|A \geq a_i) < a_i$ or (ii) $p(X|A \leq a_i) > a_i$.

Assume (i). (The proof for case (ii) is similar, so we omit it). Since $p(X|A \geq a_i) < a_i$, then $p(X|A \geq a_i) = a_i - c$ for some positive constant c . Since \mathcal{A} is finite and $a_i \neq 0$, we can choose $q_0 > a_{i-1}$ and q_1 with $a_i - c < q_0 < q_1 \leq a_i$ such that for any $q \in [q_0, q_1]$, $p(X|A > q) < q_0$.

Using similar reason as employed above to prove Fact 4, we then have for all $q \in [q_0, q_1]$:

$$p(X, A \leq q) > \pi - q_0 \cdot p(A > q) \quad (8)$$

$$p(\bar{X}, A > q) > (1 - q_0)p(A > q) \quad (9)$$

Using Lemma 2, we have:

$$\begin{aligned}
 E_p(S(A)) &= \int_0^1 q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \lambda(dq) \\
 &> \int_{q_0}^{q_1} q \cdot p(\bar{X}, A > q) + (1 - q)p(X, A \leq q) \lambda(dq) \\
 &> \int_{q_0}^{q_1} \pi(1 - q) + (q - q_0)p(A > q) \lambda(dq) \tag{\alpha}
 \end{aligned}$$

(α) follows from inequalities (8) and (9).

We now consider the difference between (α) and p 's expected self-score, i.e., $E_p(S(\pi))$. There are three cases to consider:

Case 1: $\pi < q_0$. In this case:

$$\begin{aligned}
 (\alpha) - E_p(S(\pi)) &= (\alpha) - \left(\pi \int_{\pi}^1 (1 - q) \lambda(dq) + (1 - \pi) \int_0^{\pi} q \lambda(dq) \right) \\
 &= (\alpha) - \pi \int_{\pi}^{q_0} (1 - q) \lambda(dq) - \pi \int_{q_0}^{q_1} (1 - q) \lambda(dq) \tag{10} \\
 &\quad - \pi \int_{q_0}^1 (1 - q) \lambda(dq) - (1 - \pi) \int_0^{\pi} q \lambda(dq)
 \end{aligned}$$

By Theorem 1, λ must give every non-degenerate interval positive measure, but there are no additional restrictions. Therefore, for arbitrarily small $\varepsilon > 0$, we can choose λ such that:

$$\int_0^{q_0} \lambda(dq) + \int_{q_1}^1 \lambda(dq) < \varepsilon \tag{11}$$

Since $\pi \leq 1$, and $q \leq 1$ for all values of q , the second, fourth, and fifth terms of line (10) can be made arbitrarily close to 0. We can also make $\int_{q_0}^{q_1} \lambda(dq)$ arbitrarily large. So,

$$(\alpha) - \pi \int_{q_0}^{q_1} (1 - q) \lambda(dq) = \int_{q_0}^{q_1} p(A > q) \lambda(dq) \tag{12}$$

can be made arbitrarily large since $p(A > q) > 0$. Putting these facts together, we have:

$$E_p(S(A)) - E_p(S(\pi)) > \int_{q_0}^{q_1} p(A > q) \lambda(dq) - \varepsilon \tag{13}$$

which is > 0 for some λ .

Case 2: $q_0 < \pi < q_1$. In this case, the proof is similar, except we change the bounds on the integrals and make λ large between π and q_1 , and make it arbitrarily small elsewhere.

Case 3: $q_1 < \pi$. In this case:

$$\begin{aligned}
 (\alpha) - E_p(S(\pi)) &= (\alpha) - \pi \int_{\pi}^1 (1 - q) \lambda(dq) - (1 - \pi) \int_0^{q_0} q \lambda(dq) \tag{14} \\
 &\quad - (1 - \pi) \int_{q_0}^{q_1} q \lambda(dq) - (1 - \pi) \int_{q_1}^{\pi} q \lambda(dq)
 \end{aligned}$$

We again choose λ such that

$$\int_{q_1}^1 \lambda(dq) + \int_0^{q_0} \lambda(dq) < \varepsilon$$

for arbitrarily small $\varepsilon > 0$.

With such a λ ,

$$\begin{aligned} E_p(S(A)) - E_p(S(\pi)) &> (\alpha) - E_p(S(\pi)) \\ &> \int_{q_0}^{q_1} \pi(1-q) + (q-q_0)p(A > q) - (1-\pi)q \lambda(dq) \\ &\quad - \varepsilon \\ &= \int_{q_0}^{q_1} \pi - q + (q-q_0)p(A > q) \lambda(dq) - \varepsilon \\ &> 0 \end{aligned}$$

The last inequality follows because $\pi > q_1$ and ε is sufficiently small. \square

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